

# Density-density correlation and interference mechanism for two initially independent Bose-Einstein condensates

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## Abstract

In an exciting experiment by MIT's group (Science **275**, 637 (1997)), clear interference fringes were observed for two initially independent Bose condensates in dilute gas. Presently, there are two different theories (measurement-induced interference theory and interaction-induced interference theory) which can both explain MIT's experimental results. Based on our interaction-induced interference theory, we consider the evolution of the density-density correlation after the releasing of a double-well potential trapping two independent Bose condensates. Based on the interaction-induced interference theory, we find that the interference fringes in the density-density correlation exhibit a behavior of emergence and disappearance with the development of time. We find essential difference for the density-density correlation based on interaction-induced interference theory and measurement-induced interference theory, and thus we suggest the density-density correlation to experimentally reveal further the interference mechanism for two initially independent Bose condensates.

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## I. INTRODUCTION

The interference between two initially spatially-separated Bose-Einstein condensates (BECs) is a fundamental physical problem [1]. The potential application of atom interferometry makes the theoretical and experimental researches on the interference mechanism become quite attractive. On the other hand, the theoretical and experimental researches in the last ten years show that the physical mechanism of the interference phenomena between two initially independent Bose condensates is a quite nontrivial problem. In fact, there exist considerable controversies about the physical mechanism of the interference phenomena for two initially independent Bose condensates. Presently, a popular interpretation about the interference patterns observed in [1] for two initially independent condensates is the so-called measurement-induced interference theory [2, 3, 4, 5, 6, 7]. Recently, we proposed a new interpretation of interaction-induced interference theory [8, 9, 10, 11], which agrees well with the MIT's experimental results [1]. Recently, L. S. Cederbaum *et al* [12] supported the viewpoint that interatomic interaction plays an essential role in the observed interference phenomena. Most recently, the theoretical work by D. Masiello and W. P. Reinhardt [13] also showed the emergence of interference fringes in the presence of interatomic interaction.

Considering the fact that the interference mechanism between two initially independent Bose condensates is a fundamental and subtle question, and the fact that two completely different theories (i.e. measurement-induced interference theory and interaction-induced theory) coexist, an important and urgent problem is to propose an experimental suggestion to test two different theories. In this paper, we give an experimental scheme to test two theories in future experiment based on the calculations of the density-density correlation. Different from our previous works about the interaction-induced interference theory [8, 9, 10, 11] and other works supporting this theory [12, 13], to the best of our knowledge, the present paper gives the first calculations about the density-density correlation in the frame of interaction-induced interference theory.

In MIT's experiment [1], the density distribution (first-order coherence) was measured after the overlapping between two initially independent Bose condensates. In the last few years, intensive theoretical [14, 15, 16, 17, 18, 19, 20] and experimental studies [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31] have shown that high-order correlation is very powerful in revealing the many-body quantum characteristics of ultracold atomic system. For

example, the second-order correlation function was studied experimentally in [24] to reveal whether atom lasers exhibit a truly laserlike behavior. We believe it is worthwhile to study the high-order correlation for two initially independent Bose condensates, and (up to our knowledge) the previous theoretical studies about the role of interatomic interaction in the interference mechanism did not address this sort of problem yet [8, 9, 10, 11, 12, 13]. Based on the interaction-induced interference theory, we calculate the evolution of the density-density correlation after releasing a double-well potential trapping two independent condensates. We find that the interference fringes in the density-density correlation exhibit a special behavior of emergence and disappearance with the development of time. Besides this interesting behavior, we also notice that there is an essential difference in the density-density correlation between interaction-induced interference theory and measurement-induced interference theory, and thus we suggest the experimental studies of the density-density correlation to test further the interference mechanism of two initially independent Bose condensates.

Because the interference mechanism is a quite subtle problem, we think it would be quite useful to give a brief introduction to the problem, experimental facts and different theories. About ten years ago, the MIT group gave strong evidence about the spatial coherence for Bose condensates through the observation of clear interference patterns for two separated condensates [1]. In MIT's experiment, two separated condensates were prepared in a double-well system after evaporative cooling. After releasing the double-well trapping potential, two condensates expand and overlap with sufficiently long expanding time. After their overlapping, the MIT group observed clear interference patterns for two completely different situations. (i) When the chemical potential of the system is larger than the height of the center barrier of the double-well potential, the two condensates prepared after the evaporative cooling can be regarded to be coherently separated. In this situation, every atom is described by an identical wave function which is a coherent superposition of two wave packets in different wells. It is not surprising that clear interference patterns can be observed. In fact, the MIT group did observe clear interference patterns in this situation. (ii) If the chemical potential of the system is much smaller than the height of the center barrier so that the tunneling between two wells can be omitted, after the evaporative cooling for the cold atomic cloud in the double-well trap, two condensates prepared are spatially-separated and independent. Here "independent" means that any atom either exists in the left well, or exists in the right well. It is understandable that no atom's quantum state is

a coherent superposition of the wave functions in two separated wells, because during the evaporative cooling process, the losses of the atoms and interatomic collisions lead to strong decoherence between two wells, and the negligible tunneling can not restore the coherence between two wells even at zero temperature. In this situation, the initial quantum state of the whole system at zero temperature is widely described by a Fock state  $|N_1, N_2\rangle$ , with  $N_1$  and  $N_2$  being the number of particles in the left and right wells. The most exciting phenomenon in MIT's experiment is the observation of clear interference patterns for the later situation (two initially independent condensates). For two initially independent condensates, before the releasing of the double-well potential, there is no coherence between two condensates. Exaggeratedly speaking, this is similar to the situation that there is no coherence or correlation between two condensates separately prepared in two different experimental apparatuses. For each condensate, there are no interference patterns during the free expansion. The interference patterns emerge after the overlapping between two condensates. Thus, the experimental observation of the interference patterns means that a coherence between two initially independent condensates must be established by a physical mechanism. The fundamental question is what's the physical mechanism for this coherence-establishment process.

Presently, the most popular viewpoint is the so-called measurement-induced interference theory [2, 3, 4, 5, 6, 7]. Measurement-induced interference theory thinks that before the measurement of the density distribution, there is no interference term in the density expectation value even after the overlapping between two initially independent condensates. In measurement-induced interference interpretation, it is thought that the measurement process and the interference terms in the two-particle correlation  $\langle N_1, N_2 | \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}) | N_1, N_2 \rangle$  would establish the coherence between two overlapping and initially independent condensates. When the number of particles is much larger than 1, with more and more particles being detected, the measurement induces the transformation from the Fock state to a phase state:  $|N/2, N/2\rangle \Rightarrow \left( \hat{a}_1^\dagger e^{i\varphi/2} + \hat{a}_2^\dagger e^{-i\varphi/2} \right)^N |0\rangle / (2^N N!)^{1/2}$ . Here  $\hat{a}_1^\dagger$  and  $\hat{a}_2^\dagger$  are the creation operators for the left and right condensates, and  $\varphi$  is a random phase factor. After this transformation, it is argued that there would be interference patterns in the measurement result of the density distribution. In measurement-induced interference interpretation, the interference terms in two-particle correlation function play a key role in the formation process of the phase state

(a clear introduction to this process can be found in [6]).

Recently, we found that interatomic interaction can establish the coherence between two initially independent condensates after their overlapping [8, 9, 10, 11]. In our interaction-induced interference theory [8, 9, 10], it is argued that when interatomic interaction is considered, with the evolution of the whole quantum state (the evolution of the quantum state is obtained by solving the many-body Schrödinger equation), the interference patterns emerge definitely in the density expectation value, after the overlapping between two initially independent condensates. This means that the relative phase (note that this relative phase is completely different from the insignificant relative phase between two initially independent condensates before the releasing of the double-well potential) between two condensates after the establishment of the coherence is definite, and can be known in principle when the initial condition is known. In the measurement-induced interference theory, however, the relative phase  $\varphi$  is random in every single-shot experiment. Thus, observing whether there is random relative phase (i.e. random shift of the interference patterns) in different experiments can give us direct test between measurement-induced interference theory and interaction-induced interference theory. However, more careful analyses show that this sort of experiment may be quite challenging. Even for completely identical initial condition for the cold atomic cloud in the double-well trap, after the evaporative cooling, there are unavoidable particle-number fluctuations for the condensates in two wells (i.e. for every experiment, the particle number in each well is different). Roughly speaking, if the interaction-induced interference theory is correct, the particle-number fluctuations will lead to a random relative phase approximated as  $|\Delta\mu t/\hbar|$  ( $\Delta\mu$  is the difference of the chemical potential for two condensates). If  $|\Delta\mu t/\hbar| \gtrsim \pi$ , the experimental observation of the random shift of interference patterns could not give definite evidence to distinguish the measurement-induced interference theory and interaction-induced interference theory. For the parameters in MIT's experiment, simple estimation shows that for the expansion time of  $t_{\text{exp}} = 40$  ms,  $|\Delta\mu t/\hbar| \approx 20\pi |\Delta N/N|$  [32]. Here  $\Delta N/N$  is the relative difference of the particle number in two wells. In MIT's experiment, rough estimation shows that  $|\Delta N/N|$  is larger than 2% [33]. In this situation, even random shift of the interference patterns was observed in MIT's experiment, the interaction-induced interference theory can not be excluded. Other technical noises would further challenge the test of two theories based on the measurement of the random relative phase in the density distribution. M. A. Kasevich [36] gave a very

interesting discussion about the technical noise and random phase shift by pointing out that ‘In the experiments of [1], the phase of the observed interference pattern did fluctuate, though it was not possible to unambiguously identify the mechanism of the fluctuations, because technical noise sources were also present in the measurements.’.

The above analyses show that further experimental studies are needed to reveal the interference mechanism for two initially independent condensates. In measurement-induced interference theory, when  $N_1 \gg 1$  and  $N_2 \gg 1$ , there would be clear interference fringes after the overlapping of two initially independent condensates. In our interaction-induced interference theory, even for the situation of  $N_1 \gg 1$  and  $N_2 \gg 1$ , there would be no observable interference fringes for sufficiently small interaction effect. In the extreme noninteracting situation, the interaction-induced interference theory gives a prediction that there is always no interference fringes in the density distribution for the initial Fork state  $|N_1, N_2\rangle$ . In an experiment, by adjusting the particle number, separation between two wells and even interatomic interaction through Feshbach resonance, one may test two different theories from the measurement of the density distribution.

Considering the fact that the interference mechanism is a very subtle problem, besides the test based on first-order spatial coherence, we believe high-order coherence can provide further important and at least complementary information about the interference mechanism. This is a little similar to the discrimination between a thermal light (a series of independent photons) and a laser beam based on the high-order correlation. Another merit of the density-density correlation lies in that for the experimental conditions and parameters in the original work of Ref. [1], we can give important test about the interaction-induced interference theory and measurement-induced interference theory based on the experimental studies of the density-density correlation.

We consider the following normalized density-density correlation function

$$g_{nn}(\mathbf{d}, t) = \frac{\int \langle \hat{n}(\mathbf{r} + \mathbf{d}/2, t) \hat{n}(\mathbf{r} - \mathbf{d}/2, t) \rangle dV}{\int \langle \hat{n}(\mathbf{r} + \mathbf{d}/2, t) \rangle \langle \hat{n}(\mathbf{r} - \mathbf{d}/2, t) \rangle dV}. \quad (1)$$

This sort of density-density correlation was studied recently for ultracold bosons [28, 29] and fermions [30] released from an optical lattice. Based on the interaction-induced interference theory, at the beginning of the overlapping and due to the presence of interference term in the density-density correlation  $\langle \hat{n}(\mathbf{r} + \mathbf{d}/2, t) \hat{n}(\mathbf{r} - \mathbf{d}/2, t) \rangle$ , there would be interference structure in  $g_{nn}(\mathbf{d}, t)$ . With further time evolution and when two condensates become fully

coherent, the quantum state can be approximated as  $|N\rangle$ .  $g_{nn}(\mathbf{d}, t)$  would become flat in this situation. This means that there is a behavior of emergence and disappearance for the interference fringes in  $g_{nn}(\mathbf{d}, t)$  based on the interaction-induced interference theory. In the measurement-induced interference theory, however, after the overlapping between two condensates,  $g_{nn}(\mathbf{d}, t)$  would always exhibit a flat behavior because the measurement makes the system become a phase state. These simple analyses show clearly that there is essential difference in  $g_{nn}(\mathbf{d}, t)$  between measurement-induced interference theory and interaction-induced interference theory. A merit in the measurement of the density-density correlation function is that small particle-number fluctuations during the evaporative cooling will not change significantly the structure of the density-density correlation function.

The paper is organized as follows. In Sec. II, we give the result of density-density correlation function based on the interaction-induced interference theory, while in Sec. III we give the prediction of density-density correlation function based on measurement-induced interference theory. In the last section, a brief summary and discussion are given.

## II. DENSITY-DENSITY CORRELATION FUNCTION IN INTERACTION-INDUCED INTERFERENCE THEORY

Coherence establishment mechanism due to interparticle interaction is in fact widely known and appears in a large number of physical phenomena. A famous example is the quantum phase transition from Mott insulator state to superfluid state for ultracold atoms in optical lattices [37, 38, 39], when the strength of the optical lattices is decreased adiabatically below a critical value. One should note that interatomic interaction plays key role in this quantum phase transition process. Without the interatomic interaction, decreasing the strength of the optical lattices can make the wavepacket of the atoms exist in the whole optical lattices, but all the atoms are in different and orthogonal quantum states. For the double-well system, the adiabatic varying of the center barrier has been studied intensively [40, 41, 42, 43, 44, 45]. These theoretical studies show clearly that interparticle interaction and the adiabatic decreasing of the center barrier make the initial Fock state  $|N/2, N/2\rangle$  become a coherently superposed state  $|N\rangle$  where the quantum state of all atoms become a coherent superposition of two wavepackets in different wells. Most recently, we considered the general situation with unequal particle number in two wells, and the interaction-induced

coherence process was also found, especially the Josephson effect for two initially independent Bose condensates was predicted [11]. In MIT's experiment, the double-well potential for two initially independent condensates was switched off suddenly to observe the interference patterns. This releasing of the double-well potential can be regarded as a non-adiabatic decreasing of the center barrier. Although non-adiabatic process can be quite different from an adiabatic process, the interaction-induced coherence mechanism for the adiabatic process of the double-well system gives us inspiration that interatomic interaction could play an important role in the interference phenomena observed in MIT's experiment.

In our interaction-induced interference theory [8, 9, 10], the interatomic interaction plays a role of establishing the spatial coherence between two initially independent condensates. We assume that initially there are  $N_1$  particles in the left condensate and  $N_2$  particles in the right condensate. We assume further that the single-particle wave functions are respectively  $\phi_1$  and  $\phi_2$ . After releasing the double-well potential trapping two independent condensates, even after their overlapping,  $\phi_1$  and  $\phi_2$  are always orthogonal without interatomic interaction. In the presence of interatomic interaction, however,  $\phi_1$  and  $\phi_2$  are no longer orthogonal after their overlapping. If we still use two orthogonal basis  $\phi_1$  and  $\phi'_2$ , the non-orthogonal property between  $\phi_1$  and  $\phi_2$  is equivalent to the physical picture that there are coherent particle exchanges between the orthogonal basis  $\phi_1$  and  $\phi'_2$  in the presence of interatomic interaction. We have verified rigorously that for large particle number, even a small non-orthogonality between  $\phi_1$  and  $\phi_2$  can lead to high-contrast interference fringes in the density expectation value  $\langle \hat{n}(\mathbf{r}, t) \rangle$  [8, 9, 10].

When the non-orthogonal property between  $\phi_1$  and  $\phi_2$  is considered, it is obvious that  $\hat{a}_1 = \int \phi_1^* \hat{\Psi} dV$  and  $\hat{a}_2^\dagger = \int \phi_2 \hat{\Psi}^\dagger dV$  are not commutative any more. Simple calculation gives  $[\hat{a}_1, \hat{a}_2^\dagger] = \zeta^* = \int \phi_1^* \phi_2 dV$ . In this situation, the many-body quantum state is

$$|N_1, N_2\rangle = \frac{\Xi_n}{\sqrt{N_1! N_2!}} (\hat{a}_1^\dagger)^{N_1} (\hat{a}_2^\dagger)^{N_2} |0\rangle \quad (2)$$

$\Xi_n$  is a normalization constant determined by

$$\Xi_n^2 \left( \sum_{i=0}^{N_2} \frac{N_2! (N_1 + i)! (1 - |\zeta|^2)^{N_2-i} |\zeta|^{2i}}{i! i! N_1! (N_2 - i)!} \right) = 1. \quad (3)$$

It is well-known that the field operator should be expanded in terms of a complete and orthogonal basis set. Therefore, we construct two orthogonal wave functions  $\phi_1$  and  $\phi'_2$ . Assuming that  $\phi'_2 = \beta(\phi_2 + \alpha\phi_1)$ , based on the conditions  $\int \phi_1^* \phi'_2 dV = 0$  and  $\int |\phi'_2|^2 dV = 1$ ,



we have  $|\beta| = (1 - |\zeta|^2)^{-1/2}$  and  $\alpha = -\zeta^*$ . Introducing an annihilation operator  $\hat{k} = \int \hat{\Psi}(\phi_2')^* dV$ ,  $\hat{a}_1$  and  $\hat{k}^\dagger$  are commutative. In this situation, the field operator is expanded as

$$\hat{\Psi} = \hat{a}_1 \phi_1 + \hat{k} \phi_2' + \dots \quad (4)$$

The evolution equation is obtained by first obtaining the energy expression, and then using the action principle. The overall energy is

$$E = \langle N_1, N_2, t | \hat{H} | N_1, N_2, t \rangle. \quad (5)$$

Here  $|N_1, N_2, t\rangle$  is given by Eq. (2), while  $\hat{H}$  is the Hamiltonian of the system which takes the following form

$$\hat{H} = \int dV \left( \frac{\hbar^2}{2m} \nabla \hat{\Psi}^\dagger \cdot \nabla \hat{\Psi} + \frac{g}{2} \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi} \right). \quad (6)$$

By using the ordinary action principle and the energy of the whole system, we have the following coupled evolution equations for  $\phi_1$  and  $\phi_2$  [10]:

$$i\hbar \frac{\partial \phi_1}{\partial t} = \frac{1}{N_1} \frac{\delta E}{\delta \phi_1^*}, \quad (7)$$

$$i\hbar \frac{\partial \phi_2}{\partial t} = \frac{1}{N_2} \frac{\delta E}{\delta \phi_2^*}, \quad (8)$$

where  $\delta E / \delta \phi_1^*$  and  $\delta E / \delta \phi_2^*$  are functional derivatives.

When the time evolution of  $\phi_1$  and  $\phi_2$  is obtained, the evolution of the density distribution is then

$$\begin{aligned} n(\mathbf{r}, t) &= \langle N_1, N_2, t | \hat{n} | N_1, N_2, t \rangle \\ &= \alpha_d |\phi_1(\mathbf{r}, t)|^2 + 2\beta_d \times \text{Re} (e^{i\varphi_c} \phi_1^*(\mathbf{r}, t) \phi_2(\mathbf{r}, t)) + \gamma_d |\phi_2(\mathbf{r}, t)|^2, \end{aligned} \quad (9)$$

where the coefficients are

$$\alpha_d = \sum_{i=0}^{N_2} \frac{\Xi_n^2 N_2! (N_1 + i - 1)! N_1 (1 - |\zeta|^2)^{N_2 - i} |\zeta|^{2i}}{i! i! (N_1 - 1)! (N_2 - i)!}, \quad (10)$$

$$\beta_d = \sum_{i=0}^{N_2 - 1} \frac{\Xi_n^2 N_2! (N_1 + i)! (1 - |\zeta|^2)^{N_2 - i - 1} |\zeta|^{2i + 1}}{i! (i + 1)! (N_1 - 1)! (N_2 - i - 1)!}, \quad (11)$$

$$\gamma_d = \sum_{i=0}^{N_2 - 1} \frac{\Xi_n^2 N_2! (N_1 + i)! (1 - |\zeta|^2)^{N_2 - i - 1} |\zeta|^{2i}}{i! i! N_1! (N_2 - i - 1)!}. \quad (12)$$

Here, the relative phase  $\varphi_c$  is determined by  $e^{i\varphi_c} = \zeta / |\zeta|$ . It is clearly shown that for nonzero  $\zeta$ , there is a nonzero interference pattern in  $n(\mathbf{r}, t)$ . For  $N_1 |\zeta| \gg 1$  and  $N_2 |\zeta| \gg 1$ , we have proven that  $n(\mathbf{r}, t)$  can be approximated very well as [8, 9, 10]

$$n(\mathbf{r}, t) \approx \left| \sqrt{N_1} \phi_1(\mathbf{r}, t) + \sqrt{N_2} e^{i\varphi_c} \phi_2(\mathbf{r}, t) \right|^2. \quad (13)$$

In this situation,  $\Phi_e = \sqrt{N_1} \phi_1 + \sqrt{N_2} e^{i\varphi_c} \phi_2$  can be regarded an effective order parameter of the whole system. For  $N_1 |\zeta| \gg 1$  and  $N_2 |\zeta| \gg 1$ , we have also proven that  $\Phi_e$  satisfies the ordinary Gross-Pitaevskii equation (a detailed analysis can be found in [10])

$$i\hbar \frac{\partial \Phi_e}{\partial t} \simeq -\frac{\hbar^2}{2m} \nabla^2 \Phi_e + g |\Phi_e|^2 \Phi_e. \quad (14)$$

In brief, in interaction-induced interference theory, the interatomic interaction establishes the coherence between two initially independent condensates, and thus leads to the emergence of the nonzero interference term in the density expectation value. The interference fringes based on our interaction-induced interference theory agree well with the MIT's experiment [8, 10].

After straightforward calculations, the normalized density-density correlation function is given by

$$g_{nn}(\mathbf{d}, t) = \frac{\int \langle N_1, N_2, t | \hat{n}(\mathbf{r} + \mathbf{d}/2, t) \hat{n}(\mathbf{r} - \mathbf{d}/2, t) | N_1, N_2, t \rangle dV}{\int \langle N_1, N_2, t | \hat{n}(\mathbf{r} + \mathbf{d}/2, t) | N_1, N_2, t \rangle \langle N_1, N_2, t | \hat{n}(\mathbf{r} - \mathbf{d}/2, t) | N_1, N_2, t \rangle dV} = \frac{A}{B}. \quad (15)$$

Here  $A$  and  $B$  are given in the Appendix.

Based on the expression of  $A$  and  $B$ , we have proven rigorously that  $g_{nn}(\mathbf{d}, t) \simeq 1$  for  $N_1 |\zeta| \gg 1$  and  $N_2 |\zeta| \gg 1$ . This flat behavior for  $g_{nn}(\mathbf{d}, t)$  can be understood through a simple analysis. For  $N_1 |\zeta| \gg 1$  and  $N_2 |\zeta| \gg 1$ , the quantum state (2) can be approximated well as

$$|N_1, N_2\rangle \approx |N\rangle \equiv \frac{(f^\dagger)^{N_1+N_2}}{\sqrt{(N_1+N_2)!}} |0\rangle. \quad (16)$$

Here  $f^\dagger$  is a creation operator which creates a particle in the single-particle wave function  $(\sqrt{N_1} \phi_1(\mathbf{r}, t) + \sqrt{N_2} e^{i\varphi_c} \phi_2(\mathbf{r}, t)) / \sqrt{N_1 + N_2}$ . This shows the essential physical picture that two initially independent condensates merge into a single spatially coherent condensate. For the quantum state given by Eq. (16), it is easy to prove that  $g_{nn}(\mathbf{d}, t) = 1$ .

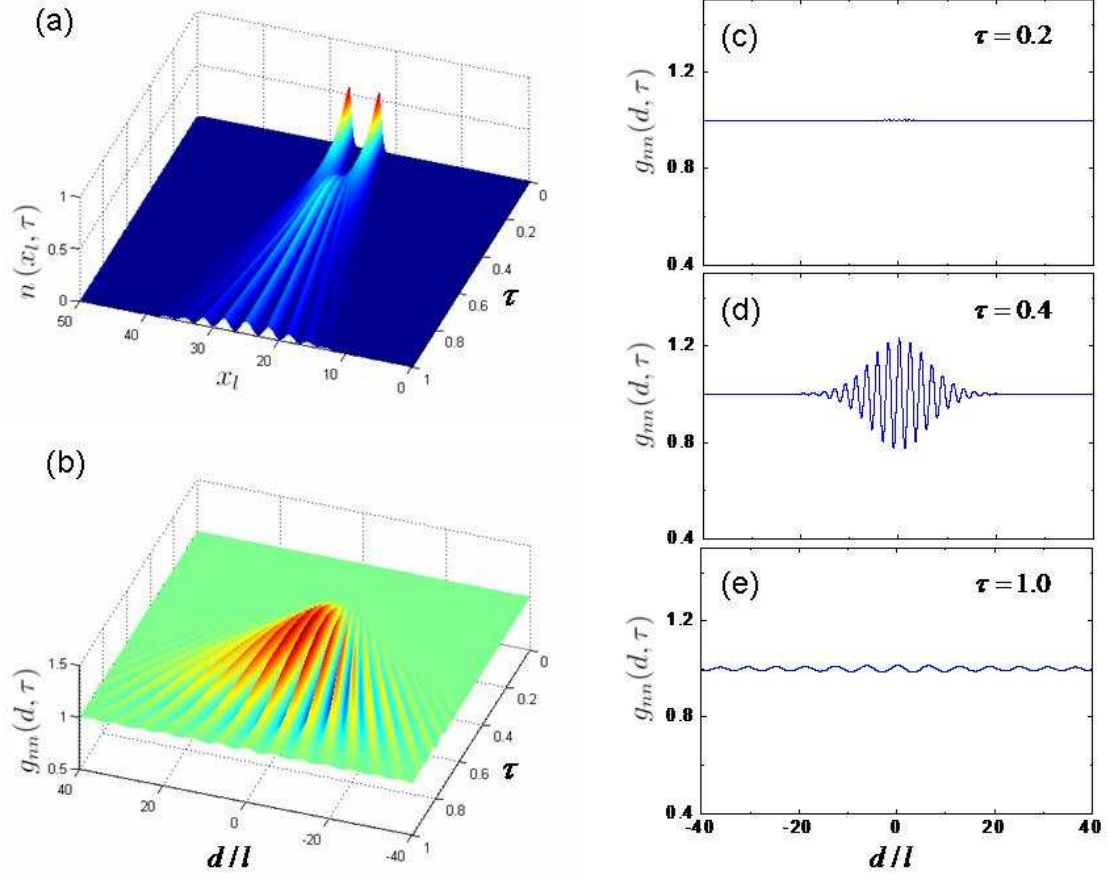


FIG. 1: (Color online) (a) gives the evolution of the density expectation value (in unit of  $N_1 + N_2$ ) which shows clear interference patterns after a full coherence between two condensates is established. (b) gives the evolution of the density-density correlation function, which shows interesting feature of emergence and disappearance of the interference patterns. (c)-(e) show further the density-density correlation function at different times.

We consider the following initial wave functions at  $t = 0$

$$\phi_1(x_l, t = 0) = \frac{1}{\pi^{1/4}\sqrt{\Delta_1}} \exp\left[-\frac{(x_l - x_{l1})^2}{2\Delta_1^2}\right] e^{i\varphi_1}, \quad (17)$$

$$\phi_2(x_l, t = 0) = \frac{1}{\pi^{1/4}\sqrt{\Delta_2}} \exp\left[-\frac{(x_l - x_{l2})^2}{2\Delta_2^2}\right] e^{i\varphi_2}. \quad (18)$$

In the above wave functions, we have introduced a dimensionless variable  $x_l = x/l$  with  $l$  being a length. The factors  $\varphi_1$  and  $\varphi_2$  are two random phases because there is no correlation between two initially independent Bose condensates. However, one can show that these

random phases play no role in the density distribution and density-density correlation. We give here an analysis for the case of the density distribution for brevity, and it is analogous for that of the density-density correlation. In the interference term (the second term) of Eq. (9),  $e^{i\varphi_c} = \zeta/|\zeta|$  with  $\zeta = \int \phi_1 \phi_2^* dV$ . From the form of the interference term, it is shown clearly that the terms  $e^{i\varphi_1}$  and  $e^{i\varphi_2}$  will be canceled out and play no role in the interference term of the density distribution.

In the numerical calculations, it is useful to introduce the dimensionless variable  $\tau = E_l t / \hbar$  with  $E_l = \hbar^2 / 2ml^2$ , and dimensionless coupling constants  $g_{l1} = N_1 g / E_l l$  and  $g_{l2} = N_2 g / E_l l$ . For the initial wave functions given by Eqs. (17) and (18), and the parameters  $\Delta_1 = \Delta_2 = 0.5$ ,  $x_{l2} - x_{l1} = 4.5$ ,  $N_1 = N_2 = 1.0 \times 10^5$ ,  $g_{l1} = g_{l2} = 20$ , the evolution of  $\phi_1$  and  $\phi_2$  is obtained numerically from Eqs. (7) and (8). The evolution of the density distribution is then given in Fig. 1(a) from the obtained  $\phi_1$ ,  $\phi_2$  and Eq. (9). With the overlapping of two condensates, clear interference patterns are shown. Fig. 1(b) gives  $g_{nn}(d, \tau)$  from obtained  $\phi_1$ ,  $\phi_2$  and Eq. (15). As shown in Fig. 1(b), interference structure in  $g_{nn}(d, \tau)$  emerges after the initial overlapping between two condensates, because two condensates are partially coherent. With further overlapping, however, the interference structure disappears because two initially independent condensates have merged into a single condensate. This unique behavior could provide definite signal in an experiment to test the interaction-induced interference theory. Figs. 1(c)-(e) show further  $g_{nn}(d, \tau)$  at different times.

Generally speaking,  $|N_1, N_2\rangle$  given by Eq. (2) can be regarded as a mixture of coherently superposed condensate and incoherent condensates. The interference structure in  $g_{nn}(d, \tau)$  originates from the incoherent component. For the incoherent component, the interference structure in  $g_{nn}(d, \tau)$  due to incoherent and overlapping condensates is not sensitive to the initial particle-number fluctuations in two wells. To understand this, one may recall in the Hanbury-Brown-Twiss stellar interferometer that the light strength fluctuations of two binary stars in different experiments are not very important [46, 47]. In addition, the component of coherently superposed condensate only contributes to a flat behavior in  $g_{nn}(d, \tau)$ . Therefore, the interesting behavior shown in Fig. 1(b) is not sensitive to the initial particle-number fluctuations in two wells.

### III. DENSITY-DENSITY CORRELATION FUNCTION IN MEASUREMENT-INDUCED INTERFERENCE THEORY

We now turn to consider the density-density correlation function based on measurement-induced interference theory. In measurement-induced interference theory, for the following quantum state of two initially independent condensates

$$|N_1, N_2\rangle = \frac{1}{\sqrt{N_1!N_2!}} \left(\hat{a}_1^\dagger\right)^{N_1} \left(\hat{a}_2^\dagger\right)^{N_2} |0\rangle, \quad (19)$$

it is assumed that after the releasing of the double-well potential and even after the overlapping between two condensates, the quantum state preserves in this sort of Fock state. In particular, it is assumed that the normalized wave functions  $\phi_1$  and  $\phi_2$  of two condensates are always orthogonal with each other. For non-interacting situation, this can be verified rigorously and easily. When  $\phi_1$  and  $\phi_2$  are orthogonal (i.e.  $[\hat{a}_1, \hat{a}_2^\dagger] = 0$ ), it is easy to show that there is no interference term in  $\langle N_1, N_2 | \hat{n}(\mathbf{r}, t) | N_1, N_2 \rangle$ . Simple calculations give

$$\langle N_1, N_2 | \hat{n}(\mathbf{r}, t) | N_1, N_2 \rangle = N_1 |\phi_1(\mathbf{r}, t)|^2 + N_2 |\phi_2(\mathbf{r}, t)|^2. \quad (20)$$

The measurement-induced interference theory tries to find out a physical mechanism for the observed interference effect missed in the above expression.

One can prove the following formula

$$|N/2, N/2\rangle = \left(\frac{\pi N}{2}\right)^{1/4} \int_0^{2\pi} \frac{d\varphi}{2\pi} |\varphi, N\rangle, \quad (21)$$

where the phase state  $|\varphi, N\rangle$  is given by

$$|\varphi, N\rangle = \frac{1}{(2^N N!)^{1/2}} \left(\hat{a}_1^\dagger e^{i\varphi/2} + \hat{a}_2^\dagger e^{-i\varphi/2}\right)^N |0\rangle. \quad (22)$$

Because for large particle number, the phase states for different phase factor  $\varphi$  can be approximated well to be orthogonal, it is argued in the measurement-induced interference theory that in a single-shot experiment, the result of the density distribution would correspond to that of a phase state with a random phase factor  $\varphi$ .

Based on the measurement-induced interference theory, after the overlapping between two condensates, the measurement makes the quantum state transform from the Fock state  $|N_1, N_2\rangle$  to a fully coherent phase state  $|\varphi, N\rangle$ . In this situation, the density-density correlation obtained after a series of experiments takes the form

$$g_{nn}(\mathbf{d}, t) = \frac{\int_0^{2\pi} d\varphi \int dV \langle \varphi, N | \hat{n}(\mathbf{r} + \mathbf{d}/2, t) \hat{n}(\mathbf{r} - \mathbf{d}/2, t) | \varphi, N \rangle}{\int_0^{2\pi} d\varphi \int dV \langle \varphi, N | \hat{n}(\mathbf{r} + \mathbf{d}/2, t) | \varphi, N \rangle \langle \varphi, N | \hat{n}(\mathbf{r} - \mathbf{d}/2, t) | \varphi, N \rangle}. \quad (23)$$

For  $N_1 \gg 1$  and  $N_2 \gg 1$  (which are satisfied for the parameters used below Eq. (18)), it is easy to find that  $g_{nn}(\mathbf{d}, t) \simeq 1$ , *i.e.*  $g_{nn}(\mathbf{d}, t)$  always shows a flat behavior. It is understandable that this flat behavior is not sensitive to the initial particle-number fluctuations in  $N_1$  and  $N_2$ .

Combined with Sec. II about the interaction-induced interference theory, we see that the behavior based on the measurement-induced interference theory (Eq. (23)) is significantly different from the prediction of the interaction-induced interference theory, where the interference fringes in the density-density correlation show a behavior of emergence and disappearance. This suggests an experimental test of the interference mechanism between the measurement-induced interference theory and interaction-induced interference theory.

#### IV. SUMMARY AND DISCUSSION

In summary, we consider the density-density correlation for two initially independent condensates based on both the interaction-induced interference theory and measurement-induced interference theory. The present work shows that there is an essential difference in the density-density correlation between two different theories. In particular, a behavior of emergence and disappearance for the interference fringes in the density-density correlation is predicted based on the interaction-induced interference theory. We suggest the studies of the density-density correlation in future experiments to test different theories, and thus deep our understanding about the interference mechanism.

As a last discussion, we consider in brief a recent interesting experiment about the vortex formation by interference of three initially independent condensates [48]. In this experiment, three initially independent condensates were first prepared in a special trapping potential which has a structure of three ‘petals’. After the decreasing of the barrier separating three condensates, the vortex was observed with a probability agreeing well with the assumption that there are indeterminate relative phases during the merging of three initially independent condensates. Although the authors of this experimental paper used the measurement-induced interference theory to explain the indeterminate relative phases and merging of three independent condensates into a single condensate, we think this experiment could be explained more naturally with interaction-induced quantum merging theory [8, 9, 10, 11]. In this experiment, the establishment of the relative phase and coherence between different con-

condensates happen before the measurement of the vortex structure, i.e. in the whole formation process of the vortex there is not a measurement at all. Therefore, it is mysterious to think that a measurement induces the relative phase and coherence between initially independent condensates in this experiment about vortex formation. In the frame of interaction-induced interference (coherence) theory, the interatomic interaction constructs the coherence and relative phase between different condensates. Together the role of damping, a stable vortex can be finally formed. In an experiment, because different condensates have different particle number and other indeterminate factors, the relative phase after the establishment of coherence is random. These analyses show that this experiment can be explained in the interaction-induced coherence theory. Further experiments of vortex formation would be quite interesting if the particle number or interatomic scattering length are controlled, so that the role of quantum merging due to interatomic interaction can be tested and studied further.

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## Appendix

In this appendix, we give the complex coefficients in  $A$  and  $B$  for  $g_{nn}(\mathbf{d}, t)$  given by Eq. (15). From the form of  $g_{nn}(\mathbf{d}, t)$ , there are 16 terms in  $A$  and  $B$ .

$$\begin{aligned}
A = & \int dV [h_1 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_2 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_3 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_4 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_5 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_6 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_7 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_8 \phi_1^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_9 \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_{10} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_{11} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_{12} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_1(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_{13} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_{14} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_1^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2) \\
& + h_{15} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_1(\mathbf{r} - \mathbf{d}/2) \\
& + h_{16} \phi_2'^*(\mathbf{r} + \mathbf{d}/2) \phi_2'(\mathbf{r} + \mathbf{d}/2) \phi_2'^*(\mathbf{r} - \mathbf{d}/2) \phi_2'(\mathbf{r} - \mathbf{d}/2)]. \tag{24}
\end{aligned}$$

The sixteen coefficients are given by



$$\begin{aligned}
h_1 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_1 + i)! (N_1 + i)^2 |\beta\zeta|^{2i}, \\
h_2 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! (N_1 + i + 1) |\beta\zeta|^{2i} \beta^* \zeta, \\
h_3 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! (N_1 + i) |\beta\zeta|^{2i} \beta \zeta^*, \\
h_4 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_2 - i) (N_1 + i)! (N_1 + i) |\beta\zeta|^{2i}, \\
h_5 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! (N_1 + i) |\beta\zeta|^{2i} \beta^* \zeta, \\
h_6 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-2} C_{N_2}^i C_{N_2}^{i+2} (N_2 - i)! (N_1 + i + 2)! |\beta\zeta|^{2i} (\beta^* \zeta)^2, \\
h_7 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i + 1)! (N_1 + i)! (N_1 + i) |\beta\zeta|^{2i}, \\
h_8 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_2 - i) (N_1 + i + 1)! |\beta\zeta|^{2i} \beta^* \zeta, \\
h_9 &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! (N_1 + i + 1) |\beta\zeta|^{2i} \beta \zeta^*, \\
h_{10} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_2 - i) (N_1 + i + 1)! |\beta\zeta|^{2i}, \\
h_{11} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-2} C_{N_2}^i C_{N_2}^{i+2} (N_2 - i)! (N_1 + i + 2)! |\beta\zeta|^{2i} (\beta \zeta^*)^2, \\
h_{12} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_2 - i - 1) (N_1 + i + 1)! |\beta\zeta|^{2i} \beta \zeta^*, \\
h_{13} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_2 - i) (N_1 + i)! (N_1 + i) |\beta\zeta|^{2i}, \\
h_{14} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_2 - i - 1) (N_1 + i + 1)! |\beta\zeta|^{2i} \beta^* \zeta, \\
h_{15} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_2 - i) (N_1 + i + 1)! |\beta\zeta|^{2i} \beta \zeta^*, \\
h_{16} &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_2 - i)^2 (N_1 + i)! |\beta\zeta|^{2i}. \tag{25}
\end{aligned}$$

$B$  also comprises sixteen coefficients.  $B$  can be obtained by replacing  $h_i$  in  $A$  by the following  $p_i$  ( $i=1, \dots, 16$ ):

$$\begin{aligned} p_1 &= a^2, p_2 = ab, p_3 = ac, p_4 = ad, p_5 = ab, p_6 = b^2, p_7 = bc, p_8 = bd, \\ p_9 &= ac, p_{10} = bc, p_{11} = c^2, p_{12} = cd, p_{13} = ad, p_{14} = bd, p_{15} = cd, p_{16} = d^2. \end{aligned} \quad (26)$$

Here

$$\begin{aligned} a &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_1 + i)! (N_1 + i) |\beta\zeta|^{2i}, \\ b &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! |\beta\zeta|^{2i} \beta^* \zeta, \\ c &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2-1} C_{N_2}^i C_{N_2}^{i+1} (N_2 - i)! (N_1 + i + 1)! |\beta\zeta|^{2i} \beta \zeta^*, \\ d &= \frac{\Xi_n^2}{N_1!N_2!|\beta|^{2N_2}} \sum_{i=0}^{N_2} C_{N_2}^i C_{N_2}^i (N_2 - i)! (N_2 - i) (N_1 + i)! |\beta\zeta|^{2i}. \end{aligned} \quad (27)$$

For  $N_1 |\zeta| \gg 1$  and  $N_2 |\zeta| \gg 1$ , we have verified by numerical calculations that  $h_i \simeq p_i$  ( $i=1, \dots, 16$ ). Therefore, we have  $g_{nn}(\mathbf{d}, t) \simeq 1$  in this situation.

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